Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Higher level
Paper 1

Thursday 6 May 2021 (afternoon)

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

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Please **do not** write on this page.

Answers written on this page
will not be marked.

Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 4]

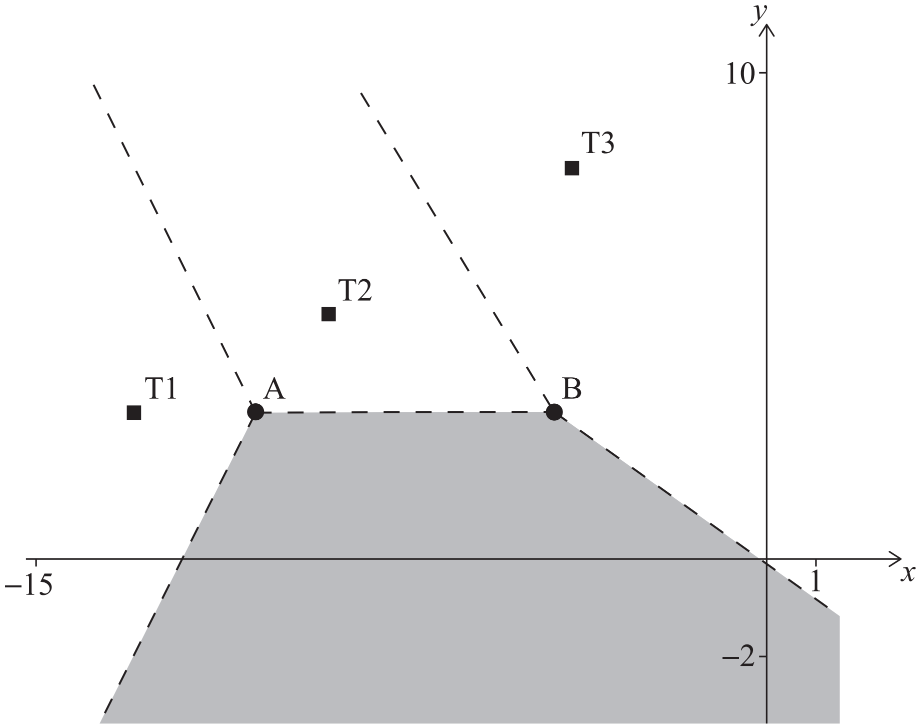
George goes fishing. From experience he knows that the mean number of fish he catches per hour is 1.1. It is assumed that the number of fish he catches can be modelled by a Poisson distribution.

On a day in which George spends 8 hours fishing, find the probability that he will catch more than 9 fish.

2. [Maximum mark: 6]

The Voronoi diagram below shows three identical cellular phone towers, $\mathrm{T1}$, $\mathrm{T2}$ and $\mathrm{T3}$. A fourth cellular phone tower, $\mathrm{T4}$ is located in the shaded region. The dashed lines in the diagram below represent the edges in the Voronoi diagram.

Horizontal scale: 1 unit represents $1\,\mathrm{km}$.
Vertical scale: 1 unit represents $1\,\mathrm{km}$.



Tim stands inside the shaded region.

(a) Explain why Tim will receive the strongest signal from tower $\mathrm{T4}$. [1]

Tower $\mathrm{T2}$ has coordinates $(-9, 5)$ and the edge connecting vertices A and B has equation $y = 3$.

(b) Write down the coordinates of tower $\mathrm{T4}$. [2]

Tower $\mathrm{T1}$ has coordinates $(-13, 3)$.

(c) Find the gradient of the edge of the Voronoi diagram between towers $\mathrm{T1}$ and $\mathrm{T2}$. [3]

(This question continues on the following page)

(Question 2 continued)

3. [Maximum mark: 8]

Charlie and Daniella each began a fitness programme. On day one, they both ran $500\,\text{m}$. On each subsequent day, Charlie ran $100\,\text{m}$ more than the previous day whereas Daniella increased her distance by 2% of the distance ran on the previous day.

(a) Calculate how far

(i) Charlie ran on day 20 of his fitness programme.

(ii) Daniella ran on day 20 of her fitness programme. [5]

On day n of the fitness programmes Daniella runs more than Charlie for the first time.

(b) Find the value of n. [3]

4. [Maximum mark: 6]

Professor Wei observed that students have difficulty remembering the information presented in his lectures.

He modelled the percentage of information retained, R, by the function $R(t) = 100\,\mathrm{e}^{-pt}$, $t \geq 0$, where t is the number of days after the lecture.

He found that 1 day after a lecture, students had forgotten $50\,\%$ of the information presented.

(a) Find the value of p. [2]

(b) Use this model to find the percentage of information retained by his students 36 hours after Professor Wei's lecture. [2]

Based on his model, Professor Wei believes that his students will always retain some information from his lecture.

(c) State a mathematical reason why Professor Wei might believe this. [1]

(d) Write down one possible limitation of the **domain** of the model. [1]

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5. [Maximum mark: 6]

A garden has a triangular sunshade suspended from three points $\mathrm{A}(2, 0, 2)$, $\mathrm{B}(8, 0, 2)$ and $\mathrm{C}(5, 4, 3)$, relative to an origin in the corner of the garden. All distances are measured in metres.

(a) (i) Find $\overrightarrow{\mathrm{CA}}$.

(ii) Find $\overrightarrow{\mathrm{CB}}$. [2]

(b) Find $\overrightarrow{\mathrm{CA}} \times \overrightarrow{\mathrm{CB}}$. [2]

(c) Hence find the area of the triangle ABC. [2]

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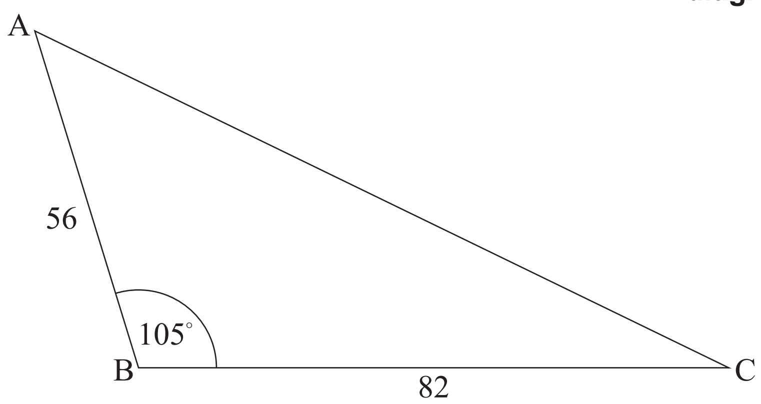
6. [Maximum mark: 5]

A triangular field ABC is such that $AB = 56\,m$ and $BC = 82\,m$, each measured correct to the nearest metre, and the angle at B is equal to 105°, measured correct to the nearest 5°.

diagram not to scale



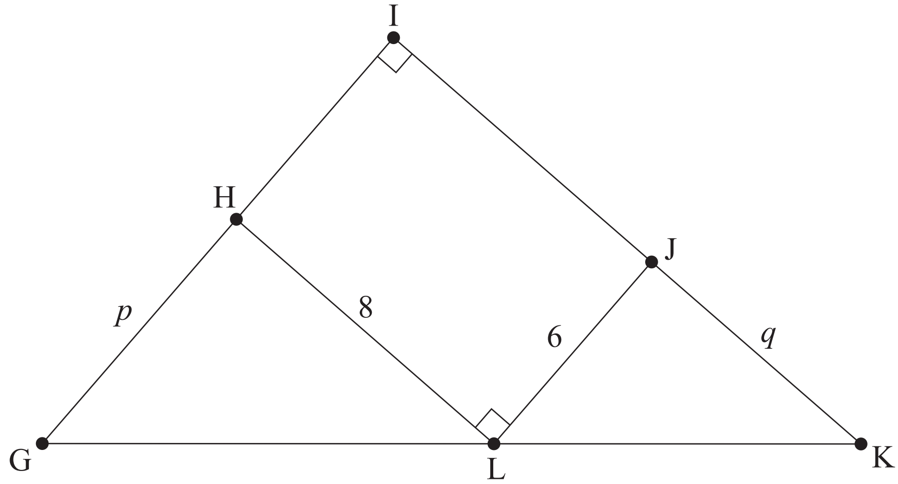
Calculate the maximum possible area of the field.

7. [Maximum mark: 8]

Ellis designs a gift box. The top of the gift box is in the shape of a right-angled triangle GIK.

A rectangular section HIJL is inscribed inside this triangle. The lengths of GH, JK, HL, and LJ are p cm, q cm, 8 cm and 6 cm respectively.

diagram not to scale



The area of the top of the gift box is $A\,\text{cm}^2$.

(a) (i) Find A in terms of p and q.

(ii) Show that $A = \dfrac{192}{q} + 3q + 48$. [4]

(b) Find $\dfrac{dA}{dq}$. [2]

Ellis wishes to find the value of q that will minimize the area of the top of the gift box.

(c) (i) Write down an equation Ellis could solve to find this value of q.

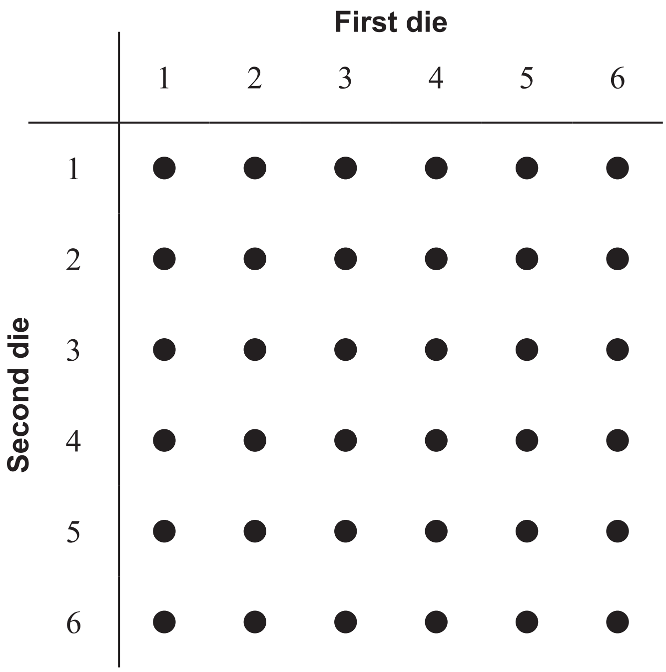
(ii) Hence, or otherwise, find this value of q. [2]

(This question continues on the following page)

(Question 7 continued)

8. [Maximum mark: 7]

A game is played where two unbiased dice are rolled and the score in the game is the greater of the two numbers shown. If the two numbers are the same, then the score in the game is the number shown on one of the dice. A diagram showing the possible outcomes is given below.



Let T be the random variable "the score in a game".

(a) Complete the table to show the probability distribution of T. [2]

t	1	2	3	4	5	6
$\mathrm{P}(T = t)$						

(b) Find the probability that

(i) a player scores at least 3 in a game.

(ii) a player scores 6, given that they scored at least 3. [3]

(c) Find the expected score of a game. [2]

(This question continues on the following page).

(Question 8 continued)

9. [Maximum mark: 8]

Consider $w = \mathrm{i}z + 1$, where $w, z \in \mathbb{C}$.

(a) Find w when

(i) $z = 2\mathrm{i}$.

(ii) $z = 1 + \mathrm{i}$. [3]

Point z on the Argand diagram can be transformed to point w by two transformations.

(b) Describe these two transformations and give the order in which they are applied. [3]

(c) Hence, or otherwise, find the value of z when $w = 2 - \mathrm{i}$. [2]

10. [Maximum mark: 7]

An engineer plans to visit six oil rigs (A–F) in the Gulf of Mexico, starting and finishing at A. The travelling time, in minutes, between each of the rigs is shown in the table.

	A	B	C	D	E	F
A		55	63	79	87	93
B	55		46	58	88	92
C	63	46		87	77	66
D	79	58	87		23	70
E	87	88	77	23		47
F	93	92	66	70	47	

The data above can be represented by a graph G.

(a) (i) Use Prim's algorithm to find the weight of the minimum spanning tree of the subgraph of G obtained by deleting A and starting at B. List the order in which the edges are selected.

(ii) Hence find a lower bound for the travelling time needed to visit all the oil rigs. [6]

(b) Describe how an improved lower bound might be found. [1]

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11. [Maximum mark: 7]

A factory, producing plastic gifts for a fast food restaurant's Jolly meals, claims that just 1% of the toys produced are faulty.

A restaurant manager wants to test this claim. A box of 200 toys is delivered to the restaurant. The manager checks all the toys in this box and four toys are found to be faulty.

(a) Identify the type of sampling used by the restaurant manager. [1]

The restaurant manager performs a one-tailed hypothesis test, at the 10% significance level, to determine whether the factory's claim is reasonable. It is known that faults in the toys occur independently.

(b) Write down the null and alternative hypotheses. [2]

(c) Find the p-value for the test. [2]

(d) State the conclusion of the test. Give a reason for your answer. [2]

12. [Maximum mark: 8]

A tank of water initially contains 400 litres. Water is leaking from the tank such that after 10 minutes there are 324 litres remaining in the tank.

The volume of water, V litres, remaining in the tank after t minutes, can be modelled by the differential equation

$$\frac{\mathrm{d}V}{\mathrm{d}t} = -k\sqrt{V}, \text{ where } k \text{ is a constant.}$$

(a) Show that $V = \left(20 - \frac{t}{5}\right)^2$. [6]

(b) Find the time taken for the tank to empty. [2]

13. [Maximum mark: 7]

A submarine is located in a sea at coordinates $(0.8, 1.3, -0.3)$ relative to a ship positioned at the origin O. The x direction is due east, the y direction is due north and the z direction is vertically upwards.

All distances are measured in kilometres.

The submarine travels with direction vector $\begin{pmatrix} -2 \\ -3 \\ 1 \end{pmatrix}$.

(a) Assuming the submarine travels in a straight line, write down an equation for the line along which it travels. [2]

The submarine reaches the surface of the sea at the point P.

(b) (i) Find the coordinates of P.

(ii) Find OP. [5]

14. [Maximum mark: 6]

The weights of apples from Tony’s farm follow a normal distribution with mean $158\,\text{g}$ and standard deviation $13\,\text{g}$. The apples are sold in bags that contain six apples.

(a) Find the mean weight of a bag of apples. [2]

(b) Find the standard deviation of the weights of these bags of apples. [2]

(c) Find the probability that a bag selected at random weighs more than $1\,\text{kg}$. [2]

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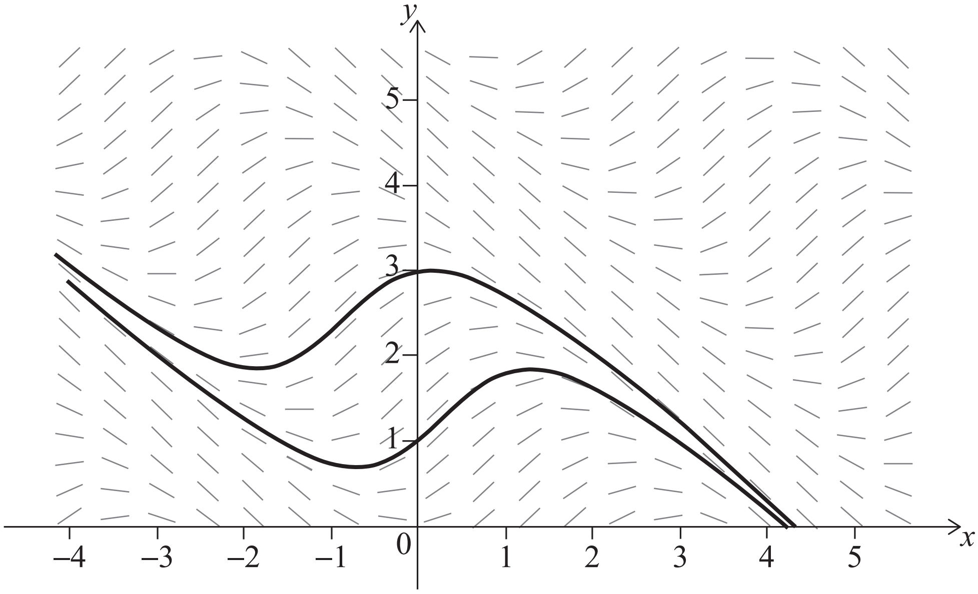
15. [Maximum mark: 5]

The diagram shows the slope field for the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \sin(x + y),\ -4 \le x \le 5,\ 0 \le y \le 5.$$

The graphs of the two solutions to the differential equation that pass through points $(0, 1)$ and $(0, 3)$ are shown.



For the two solutions given, the local minimum points lie on the straight line L_1.

(a) Find the equation of L_1, giving your answer in the form $y = mx + c$. [3]

For the two solutions given, the local maximum points lie on the straight line L_2.

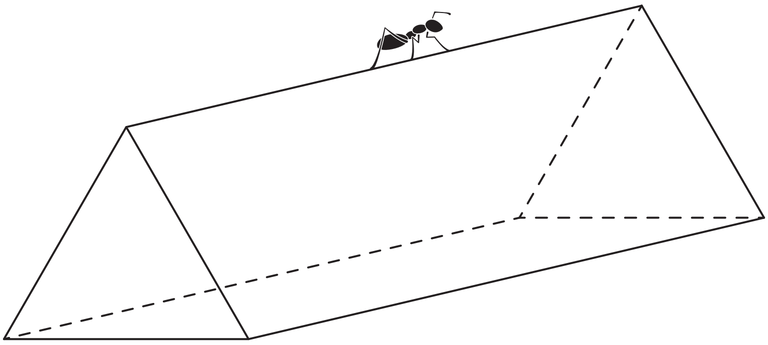
(b) Find the equation of L_2. [2]

(This question continues on the following page)

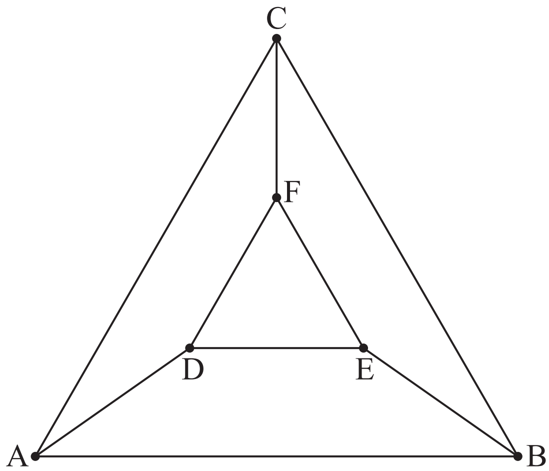
(Question 15 continued)

16. [Maximum mark: 5]

An ant is walking along the edges of a wire frame in the shape of a triangular prism.



The vertices and edges of this frame can be represented by the graph below.



(a) Write down the adjacency matrix, $\boldsymbol{M}$, for this graph. [3]

(b) Find the number of ways that the ant can start at the vertex A, and walk along exactly 6 edges to return to A. [2]

(This question continues on the following page)

(Question 16 continued)

17. [Maximum mark: 7]

The graph of the function $f(x) = \ln x$ is translated by $\begin{pmatrix} a \\ b \end{pmatrix}$ so that it then passes through the points $(0, 1)$ and $(e^3, 1 + \ln 2)$.

Find the value of a and the value of b.

References: